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Quantum Superimposing Multiple Anti-Cloning Machine

Yan-ling Li · Jian Feng · Bao-long Liang · Ya-fei Yu

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Abstract We prove that the nonorthogonal states randomly selected from a set can evolve into a linear superposition of multiple copies of anti-cloned state (an orthogonal state along with the original) with failure branch if and only if the input states are linearly independent. We derive a bound on the success probabilities of our machine. We show that probabilistic anti-cloning and multiple anti-cloning machines are special cases of our machine. The results for a single input state are also generalized into the case of several input copies of a state.

Keywords Multiple anti-cloning · Anti-cloning

1 Introduction

The quantum mechanical principles have been used to realize quantum computation, quantum teleportation, quantum key distribution and so on [13] which cannot be realized in classical world. However, these principles not only enrich quantum information but also put on some limitations on manipulations with quantum information. So there are many quantum "no-go" theorems [7, 8, 11, 14, 21]. For example, the quantum no-flipping theorem, which asserts that one cannot design a quantum flipping machine which outputs the exact orthogonal complementing state for the input of an arbitrary qubit [1, 3–5, 10], the quantum

Y.-l. Li (✉) · J. Feng · B.-l. Liang
School of Physics Science and Information Engineering, Liaocheng University, Liaocheng 252059, Shandong, China
e-mail: liyl423@yahoo.com.cn

Y.-l. Li · J. Feng · B.-l. Liang
Institute of Optical Communication, Liaocheng University, Liaocheng 252059, Shandong, China

Y.-f. Yu
Laboratory of Photonic Information Technology, South China Normal University, GuangZhou 510006, Guangdong, China

no-complementing theorem, which establishes the impossibility of producing an exact orthogonal state along with the original starting from a single copy and reflects the quantum no-anti-cloning theorem [1, 10, 18]. On the other hand, with the great advances in quantum information theory, surprising effects are continuously being discovered. For instance, in many physical situation the cloning operation and complementing transformation (i.e. the universal NOT gate) are deeply interconnected [1], these two processes are always realized contextually and their optimal fidelity are directly related [12, 20]. Another interesting related observation is that the two anti-parallel spin state contains more information than two parallel spin state [10], i.e., one can measure the spin direction of $|\psi\rangle$ with better fidelity when two qubits are in anti-parallel spin state $|\psi,\psi^{\perp}\rangle$ than in parallel one $|\psi,\psi\rangle$. The importance of considering quantum spin-flip machine and anti-cloning device is emphasized by these interesting observations.

Though perfect operations are not possible, one can realized these impossible operations in a probabilistic but exact way or a deterministic but inaccurate way [2, 6, 9, 15–19]. As for quantum anti-cloning, the probabilistic quantum anti-cloning machine, which exactly produces the output of an orthogonal state along with the original with certain success probability for the input of an unknown state, has been proposed in references [17, 18]. We notice that in the above process only a single state is produced probabilistically, i.e., $|\psi\rangle \rightarrow |\psi\rangle|\psi^{\perp}\rangle$ is merely generated with certain probabilities. However, in quantum world, one can have linear superposition of all possibilities with appropriate probabilities [15]. If a real quantum anti-cloning machine existed, it would take advantage of this basic quantum property and produce simultaneously $|\psi\rangle \rightarrow |\psi\rangle|\psi^{\perp}\rangle, |\psi\rangle \rightarrow (|\psi\rangle|\psi^{\perp}\rangle)^{\otimes 2}, \ldots, |\psi\rangle \rightarrow (|\psi\rangle|\psi^{\perp}\rangle)^{\otimes M}$, where M is a positive number. Motivated by these, it is naturally desirable to ask whether can there exist a quantum machine which takes an unknown input state and produces an output state whose success branch exists in a superposition of multiple anti-cloned states and the failure branch exists in a superposition of composite state independent of the input state? The answer is positive. The existence of such a machine is proved and a bound on the success probability of this machine is derived in Sect. 2. The generality of our machine, and the generalization to the case of several input copies of a state are discussed in Sect. 3. Our summary is presented in Sect. 4.

2 Quantum Superimposing Multiple Anti-Cloning Machine

The existence of the machine, which takes an unknown input state and produces an output state whose success branch exists in a superposition of multiple anti-cloned states and the failure branch exists in a superposition of composite state independent of the input state, can be proved by the following theorem.

Theorem 1 *There exists a unitary operator U such that for any unknown state chosen from a set $S = \{|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_k\rangle\}$ the machine can create a linear superposition of multiple anti-cloned states together with failure branch given by*

$$U(|\psi_i\rangle|\Sigma\rangle|P\rangle) = \sum_{n=1}^{M} \sqrt{p_n^{(i)}}(|\psi_i\rangle|\psi_i^{\perp}\rangle)^{\otimes n}|0\rangle^{2(M-n)}|P_n\rangle + \sum_{l=M+1}^{N_C} \sqrt{f_l^{(i)}}|\phi_l\rangle_{AB}|P_l\rangle, \quad (1)$$

if and only if the states $|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_k\rangle$ are linearly independent. The machine is named as quantum superimposing multiple anti-cloning machine.

In (1), the unknown input state $|\psi_i\rangle$ from a set S belongs to a Hilbert space $\mathsf{H}_A = C^{N_A}$; $|\Sigma\rangle$ is the state of the ancillary system B belonging to a Hilbert space H_B of dimension $N_B = N_A^{(2M-1)}$, where $(2M-1)$ is the total number of blank states each having dimension N_A; $|\psi_i^{\perp}\rangle$ is the state orthogonal to $|\psi_i\rangle$. If (1) holds, the probe P is measured after the evolution. The output states are reserved if and only if the measurement results of the probe are $|P_n\rangle$ $(n = 1, \dots, M)$. So p_n^i is the success probability for the ith input state $|\psi_i\rangle$ to produce n multiple exact anti-cloned states $(|\psi_i\rangle|\psi_i^{\perp}\rangle)^{\otimes n}$; $f_l^{(i)}$ is the failure probability for the ith input state to remain in the lth failure component. $\sum_l \sqrt{f_l^{(i)}}|\phi_l\rangle_{AB}|P_l\rangle$ represents the failure branch. The states $|\phi_l\rangle_{AB}$ $(l = M+1, M+2, \dots, N_C)$ are normalized states of the composite system AB and they are not necessarily orthogonal. $|P\rangle, |P_1\rangle, |P_2\rangle, \dots, |P_{N_C}\rangle$ are orthonormal basis states of the probing device with $N_C > M$. To prove that (1) holds with positive probability p_n^i, we introduce the following lemmas.

Lemma 1 *If the set* $S = \{|\psi_1\rangle, |\psi_2\rangle, \dots, |\psi_k\rangle\}$ *is linearly independent, then the set* $S = \{(|\psi_1\rangle|\psi_1^{\perp}\rangle)^{\otimes n}, (|\psi_2\rangle|\psi_2^{\perp}\rangle)^{\otimes n}, \dots, (|\psi_k\rangle|\psi_k^{\perp}\rangle)^{\otimes n}\}$ *is linearly independent, where n is a positive integer.*

Proof We use the "negative approach" to prove Lemma 1. Let's suppose that the set $S = \{(|\psi_1\rangle|\psi_1^{\perp}\rangle)^{\otimes n}, (|\psi_2\rangle|\psi_2^{\perp}\rangle)^{\otimes n}, \dots, (|\psi_k\rangle|\psi_k^{\perp}\rangle)^{\otimes n}\}$ is linearly dependent. Then there exists

$$\sum_{i=1}^{k} c_i(|\psi_i\rangle|\psi_i^{\perp}\rangle)^{\otimes n} = 0 \tag{2}$$

with c_i $(i = 1, 2, \dots, k)$ being not all zero. For an arbitrary unknown state $|\psi\rangle$ there exists $K|\psi\rangle = |\psi^{\perp}\rangle$, where K is an antiunitary operator [1]. Equation (2) is reduced to

$$K^n \sum_{i=1}^{k} c_i |\psi_i\rangle^{\otimes 2n} = 0 \quad (n \text{ being an even number}), \tag{3a}$$

$$K^n \sum_{i=1}^{k} c_i^* |\psi_i\rangle^{\otimes 2n} = 0 \quad (n \text{ being an odd number}). \tag{3b}$$

From (3) we obtain that $\sum_{i=1}^{k} c_i|\psi_i\rangle^{\otimes 2n} = 0$ or $\sum_{i=1}^{k} c_i^*|\psi_i\rangle^{\otimes 2n} = 0$ for each allowed positive integer n, which means that the set $S = \{|\psi_1\rangle^{\otimes 2n}, |\psi_2\rangle^{\otimes 2n}, \dots, |\psi_k\rangle^{\otimes 2n}\}$ is linearly dependent.

However, it is easy to prove that the set $S = \{|\psi_1\rangle^{\otimes n_1}, |\psi_2\rangle^{\otimes n_1}, \dots, |\psi_k\rangle^{\otimes n_1}\}$ $(n_1 \geq 1)$ is linearly independent if the set $S = \{|\psi_1\rangle, |\psi_2\rangle, \dots, |\psi_k\rangle\}$ is linearly independent. Therefore, it is in contradiction with the result of above assumption, so the set $S = \{(|\psi_1\rangle|\psi_1^{\perp}\rangle)^{\otimes n}, (|\psi_2\rangle|\psi_2^{\perp}\rangle)^{\otimes n}, \dots, (|\psi_k\rangle|\psi_k^{\perp}\rangle)^{\otimes n}\}$ is linearly independent. □

Lemma 2 *If the set* $S = \{|\psi_1\rangle, |\psi_2\rangle, \dots, |\psi_k\rangle\}$ *is linearly independent then the matrix* $X = [\langle\psi_i|\psi_j\rangle^n \langle\psi_i^{\perp}|\psi_j^{\perp}\rangle^n]$ *is positive definite, where n is a positive integer.*

Proof For an arbitrary column vector $\alpha = \mathrm{col}(c_1, c_2, \dots, c_k)$, the quadratic form $\alpha^+ X\alpha$ can be expressed as

$$\alpha^+ X\alpha = \sum_{i,j=1}^{k} c_i^* c_j X_{ij} = \langle\beta|\beta\rangle \tag{4}$$

where $|\beta\rangle = \sum_i c_i|\psi_i\rangle^{\otimes n}|\psi_i^{\perp}\rangle^{\otimes n}$. Since $|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_k\rangle$ are linearly independent, according to Lemma 1, the summation state $|\beta\rangle$ does not reduce to zero for any k vector α, and its norm $\langle\beta|\beta\rangle$ is therefore always positive. By definition (4), we conclude that the matrix X is positive definite. □

Proof of Theorem 1 Now we prove Theorem 1 in two stages. First, we show that if such a machine described by (1) exists, then $|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_k\rangle$ are linearly independent. Suppose that the states $|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_k\rangle$ are linearly dependent such that $|\psi_j\rangle = \sum_{i=1,i\neq j}^{k} c_i|\psi_i\rangle$. If we send this state through the machine defined by (1), the unitary evolution will yield

$$U(|\psi_j\rangle|\Sigma\rangle|P\rangle) = \sum_{n=1}^{M}\sqrt{p_n^{(j)}}(|\psi_j\rangle|\psi_j^{\perp}\rangle)^{\otimes n}|0\rangle^{2(M-n)}|P_n\rangle + \sum_{l=M+1}^{N_C}\sqrt{f_l^{(j)}}|\phi_l\rangle_{AB}|P_l\rangle. \quad (5)$$

However, by linearity of quantum theory each of $|\psi_i\rangle$ will go through the transformation (1) and we obtain

$$U\left(\sum_{i=1,i\neq j}^{k} c_i|\psi_i\rangle|\Sigma\rangle|P\rangle\right) = \sum_{i=1,i\neq j}^{k} c_i \sum_{n=1}^{M}\sqrt{p_n^{(i)}}(|\psi_i\rangle|\psi_i^{\perp}\rangle)^{\otimes n}|P_n\rangle + \sum_{i=1,i\neq j}^{k} c_i \sum_{l=M+1}^{N_C}\sqrt{f_l^{(i)}}|\phi_l\rangle_{AB}|P_l\rangle. \quad (6)$$

Because the final states in (5) and (6) are different, the quantum state $|\psi_j\rangle$ cannot be probabilistically anti-cloned by a general unitary-reduction operation. Therefore, linearity of quantum theory prohibits us from producing linear superposition of multiple anti-cloned states for the input states chosen from a linearly dependent set. Consequently, (1) exists for any state randomly selected from S only if the set $S = \{|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_k\rangle\}$ is linearly independent.

Conversely, we demonstrate that the linear independence of $\{|\psi_i\rangle\}$ $(i = 1, 2, \ldots, k)$ results in the existence of a unitary operator U given by (1). If the unitary evolution (1) holds, then the inner products of (1) results in the following equation

$$\langle\psi_i|\psi_j\rangle = \sum_{n=1}^{M}\sqrt{p_n^{(i)}}\langle\psi_i|\psi_j\rangle^n\langle\psi_i^{\perp}|\psi_j^{\perp}\rangle^n\sqrt{p_n^{(j)}} + \sum_{l=M+1}^{N_C}\sqrt{f_l^{(i)}f_l^{(j)}} \quad (i, j = 1, 2, \ldots, k). \quad (7)$$

Equation (7) can be denoted by the $k \times k$ matrix equation

$$G^{(1)} = \sum_{n=1}^{M} A_n X A_n^{+} + \sum_{l=M+1}^{N_C} F_l \quad (8)$$

where the matrices $G^{(1)} = [\langle\psi_i|\psi_j\rangle]$, $X = [\langle\psi_i|\psi_j\rangle^n\langle\psi_i^{\perp}|\psi_j^{\perp}\rangle^n]$, $A_n = A_n^{+} = \mathrm{diag}(\sqrt{p_n^{(1)}}, \sqrt{p_n^{(2)}}, \ldots, \sqrt{p_n^{(k)}})$ and $F_l = [\sqrt{f_l^{(i)}f_l^{(j)}}]$.

On the other hand, if we let

$$
\begin{aligned}
&|\mu_i\rangle = |\psi_i\rangle|\Sigma\rangle|P\rangle,\\
&|\tilde{\mu}_i\rangle = \sum_{n=1}^{M}\sqrt{p_n^{(j)}}(|\psi_j\rangle|\psi_j^{\perp}\rangle)^{\otimes n}|0\rangle^{2(M-n)}|P_n\rangle + \sum_{l=M+1}^{N_C}\sqrt{f_l^{(j)}}|\phi_l\rangle_{AB}|P_l\rangle \qquad (9)\\
&\quad (i, j = 1, 2, \ldots, k).
\end{aligned}
$$

Noticing that $|P\rangle, |P_1\rangle, |P_2\rangle, \ldots, |P_{N_C}\rangle$ are orthonormal basis states of the probe device, from (9) we have

$$
\begin{aligned}
&\langle\mu_i|\mu_j\rangle = \langle\psi_i|\psi_j\rangle,\\
&\langle\tilde{\mu}_i|\tilde{\mu}_j\rangle = \sum_{n=1}^{M}\sqrt{p_n^{(i)}}\langle\psi_i|\psi_j\rangle^n\langle\psi_i^{\perp}|\psi_j^{\perp}\rangle^n\sqrt{p_n^{(j)}} + \sum_{l=M+1}^{N_C}\sqrt{f_l^{(i)}f_l^{(j)}} \qquad (10)\\
&\quad (i, j = 1, 2, \ldots, k).
\end{aligned}
$$

If (8) holds, we can have that $\langle\mu_i|\mu_j\rangle = \langle\tilde{\mu}_i|\tilde{\mu}_j\rangle$. According to Lemma 1 given in reference [6], there will exist a unitary operator U satisfying $|\tilde{\mu}_i\rangle = U|\mu_i\rangle$, i.e., there exists a unitary operator U satisfying (1) if (8) holds. So we aim to prove that if the set $S = \{|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_k\rangle\}$ is linearly independent, then (8) holds for a positive definite matrix A_n.

If the nonorthogonal states $\{|\psi_i\rangle\}$ $(i = 1, 2, \ldots, k)$ are linearly independent, the matrix $G^{(1)}$ is positive definite [6] and X is also a positive definite matrix according to Lemma 2. From continuity, for small enough but positive $p_n^{(i)}$ $(i = 1, 2, \ldots, k)$ the matrix $G^{(1)} - \sum_{n=1}^{M} A_n X A_n^+$ is also a positive definite matrix. Therefore, we can diagonalize the Hermitian matrix $G^{(1)} - \sum_{n=1}^{M} A_n X A_n^+$ by a suitable unitary operator V as follows:

$$
V^+\left(G^{(1)} - \sum_{n=1}^{M} A_n X A_n^+\right)V = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_k) \qquad (11)
$$

where the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_k$ are positive real numbers. In (8), we can choose

$$
F_l = V\,\mathrm{diag}(t_l^{(1)}, t_l^{(2)}, \ldots, t_l^{(k)})V^+ \qquad (12)
$$

such that

$$
\sum_{l=M+1}^{N_C} t_l^{(i)} = \lambda_i \quad (i = 1, 2, \ldots, k). \qquad (13)
$$

Equations (11–13) indicate that (8) is satisfied with a positive matrix A_n if the states are linearly independent. This completes the proof of Theorem 1. □

Now we derive a bound on the success probabilities of this unitary transformation. Taking the inner product of two distinct states given by (1) and using relation $|\langle\psi_i|\psi_j\rangle| = |\langle\psi_i^{\perp}|\psi_j^{\perp}\rangle|$, we have

$$
|\langle\psi_i|\psi_j\rangle| \le \sum_{n=1}^{M}\sqrt{p_n^{(i)}}|\langle\psi_i|\psi_j\rangle|^{2n}\sqrt{p_n^{(j)}} + \sum_{l=M+1}^{N_C}\sqrt{f_l^{(i)}f_l^{(j)}}. \qquad (14)
$$

By using the condition of normalization

$$\sum_{n=1}^{M} p_n^{(i)} + \sum_{l=M+1}^{N_C} f_l^{(i)} = 1, \tag{15}$$

we can obtain

$$\frac{1}{2}\sum_{n=1}^{M}(p_n^{(i)} + p_n^{(j)})(1 - |\langle\psi_i|\psi_j\rangle|^{2n}) \leq 1 - |\langle\psi_i|\psi_j\rangle|. \tag{16}$$

The inequality (16) indicates that the sum of success probabilities of two distinct multiple anti-cloned states is always bounded by M and inner product of two corresponding input nonorthogonal states.

3 Discussion and Generalization

In this section, we first discuss the generality of our machine.

If the states $|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_k\rangle$ are orthogonal and all success probabilities $p_n^{(i)}$'s are nonzero, then the unitary evolution (1) allows us to have a linear superposition of multiple anti-cloned states since the matrix equation (8) is always satisfied.

If we take that $M = 1$ and all $f_l^{(i)}$'s are zero, from the proof of Theorem 1, we have that (8) cannot be satisfied for nonorthogonal states $|\psi_i\rangle$ $(i = 1, 2, \ldots, k)$ namely the quantum no-complementing theorem [1, 10, 18].

If we take that $M = 1$, then (1) reduces to

$$U(|\psi_i\rangle|\Sigma\rangle|P\rangle) = \sqrt{p_n^{(i)}}|\psi_i\rangle|\psi_i^{\perp}\rangle|P_1\rangle + \sum_l \sqrt{f_l^{(i)}}|\phi_l\rangle_{AB}|P_l\rangle \tag{17}$$

which describes a probabilistic quantum anti-cloning machine discussed in references [17, 18]. The bound on the success probabilities (16) reduces to

$$\frac{1}{2}(p_1^{(i)} + p_1^{(j)}) \leq 1/(1 + |\langle\psi_i|\psi_j\rangle|). \tag{18}$$

If we take that $n = M$, $p_M^{(i)}$ $(i = 1, 2, \ldots, k)$ are non-zero and all others are zero, then (1) reduces to

$$U(|\psi_i\rangle|\Sigma\rangle|P\rangle) = \sqrt{p_M^{(i)}}(|\psi_i\rangle|\psi_i^{\perp}\rangle)^{\otimes M}|P_M\rangle + \sum_l \sqrt{f_l^{(i)}}|\phi_l\rangle_{AB}|P_l\rangle, \tag{19}$$

which can be regarded as the probabilistic multiple anti-cloning machine. Our bound (16) reduces to

$$\frac{1}{2}(p_M^{(i)} + p_M^{(j)}) \leq (1 - |\langle\psi_i|\psi_j\rangle|)/(1 - |\langle\psi_i|\psi_j\rangle|^{2M}). \tag{20}$$

Above analyses show that, to some degree, our machine given in Theorem 1 is general.

However, this machine is limited to taking a single copy of $|\psi_i\rangle$ as input state. Then can there exist a quantum machine which takes m (m is positive integer and $m > 1$) copies of $|\psi_i\rangle$ and produces a linear superposition of multiple exact anti-cloned states with certain success probabilities? The answer is "yes" and the generalization of the superimposing multiple anti-cloning machine for a single input copy $|\psi_i\rangle$ to the one for m input copies $|\psi_i\rangle^{\otimes m}$ is straightforward, it can be completed by a way similar to the proof of Theorem 1. For the sake of conciseness, we omit its proof. The results are displayed as follows.

Theorem 2 *For any unknown nonorthogonal states chosen from a set* $S = \{|\psi_1\rangle, |\psi_2\rangle, \dots, |\psi_k\rangle\}$, *there exists a unitary operator* U *such that* $|\psi_i\rangle^{\otimes m}$ *can be evolved into a linear superposition of multiple anti-cloned states with failure branch, which is described by*

$$U(|\psi_i\rangle^{\otimes m}|\Sigma_m|P\rangle) = \sum_{n=1}^{M}\sqrt{p_n^{(i)}}(|\psi_i\rangle|\psi_i^{\perp}\rangle)^{\otimes n}|0\rangle^{2(M-n)} + \sum_{l=M+1}^{N_C}\sqrt{f_l^{(i)}}|\phi_l\rangle_{AB}|P_l\rangle \tag{21}$$

if and only if the states $|\psi_1\rangle, |\psi_2\rangle, \dots, |\psi_k\rangle$ *are linearly independent.*

In (21), $|\Sigma_m\rangle$ is the state of the ancillary system B belonging to Hilbert space H_B of dimension $N_B = N_A^{(2M-m)}$, where M, m are positive integers and $2M > m$, $(2M - m)$ is the total number of blank states each having dimension N_A. The $p_n^{(i)}$ and $f_l^{(i)}$ denote the success and failure probabilities for the ith input state $|\psi_i\rangle^{\otimes m}$ to produce n exact anti-cloned states $(|\psi_i\rangle|\psi_i^{\perp}\rangle)^{\otimes n}$. And $\sum_{n=1}^{M} p_n^{(i)} + \sum_{l=M+1}^{N_c} f_l^{(i)} = 1$. The other quantities have the same meaning as explained above.

The bound on the success probabilities of our general machine (21) is

$$\frac{1}{2}\sum_{n=1}^{M}(p_n^{(i)} + p_n^{(j)})(1 - |\langle\psi_i|\psi_j\rangle|^{2n}) \le 1 - |\langle\psi_i|\psi_j\rangle|^{m}. \tag{22}$$

The inequality (22) indicates that the sum of success probabilities of two distinct multiple anti-cloned states is always bounded by M, m and inner product of two corresponding input nonorthogonal states.

The machine described by (21) takes m copies of $|\psi_i\rangle$ and produces an output state whose success branch is the linear superposition of n exact copies of anti-cloned states $|\psi_i\rangle|\psi_i^{\perp}\rangle$ and the failure branch is the superposition of composite states independent of the input state. It is clear that Theorem 1 is a special case of Theorem 2 with $m = 1$.

4 Summary

In conclusion, we have proposed a quantum superimposing multiple anti-cloning machine. We prove that the nonorthogonal states randomly selected from a set $S = \{|\psi_1\rangle, |\psi_2\rangle, \dots, |\psi_k\rangle\}$ can be evolved into a linear superposition of multiple anti-cloning states with failure branch described by a composite state (independent of the input state) by a unitary evolution together with a measurement if and only if the states $|\psi_1\rangle, |\psi_2\rangle, \dots, |\psi_k\rangle$ are linearly independent. We derive a bound on the success probabilities of our machine. We show that probabilistic anti-cloning machine [17, 18] and multiple anti-cloning machine are special cases of our machine, the quantum "no-complementing" theorem can also be obtained from our machine. These results for a single input state have been generalized into the case of several copies of an input state. Our result may have potential applications in quantum information processing (such as quantum state engineering, anti-parallel storage of quantum information, etc.) because it provides an intrinsic regularity of quantum states in quantum computer. It also tells us how to control the success probability for the input states to produce multiple anti-cloned states in a desired way by using controllable operations.

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